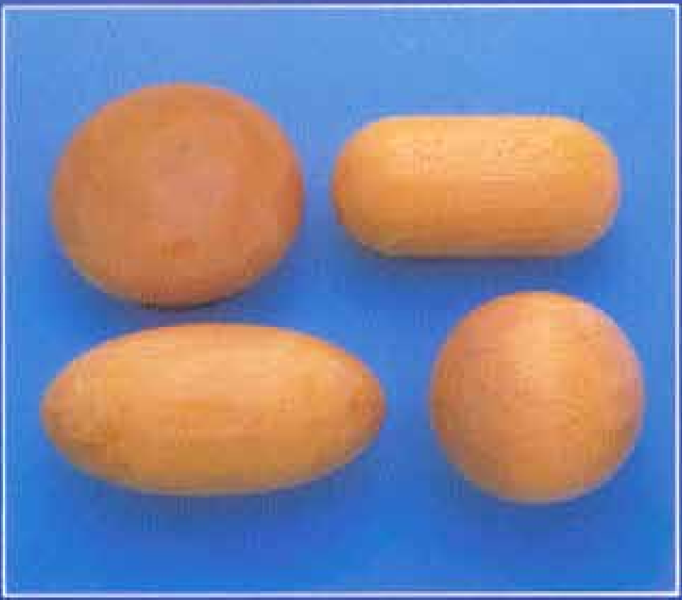
Which is the best shape for a pendulum bob? All of these shapes have the same mass and volume as the sphere and have simple ratios of length to diameter.



Above: Sphere - 2:1 prolate spheroid; 1:2 oblate spheroid; 2:1 cylinder with hemispherical ends.

Above: Parabolic spindles - 2:1, 3:1, and 4:1.

Below: Lenticular - 1:2 thickness to diameter, 1:3, 1:2 double truncated cone (after Riefler); 1:4 lenticular.

Below: Circular cylinders - 1:4, 1:3, 1:2, 1:1, 2:1, 3:1, 2:1 cylinder with one hemispherical end (after pendulum for Big Ben).

Is your bob in *better* shape?

Douglas A Bateman replies to Charles K Aked's article in last month's issue.

CHARLES Aked is wrong. Wrong in both his aerodynamic assumptions and conclusions about the best shape for a pendulum bob. In the May issue of *Clocks*, Charles Aked has responded to a short letter of mine about the spherical bob with a lengthy article on bob shapes entitled 'Is your bob in good shape?' However, the article contains many practical points on other aspects of horology and is having the effect of urging me into a belated description of some unique experiments. What follows is a summary of the results of the experiments to which Charles Aked referred in complimentary terms (well received). I will then compare our respective conclusions.

The Bateman experiments

For pendulums, improvements are possible by two direct methods. One is to increase the density of the bob for given volume to increase the stored energy, and the other is to decrease the air resistance. Both methods improve the ratio of stored energy to the energy lost per period - in other words the quality factor, Q. Both changes will be seen as common sense as, if nothing else, reducing the air resistance could reduce the driving weight and stress on a train!

Because the end results must relate to real pendulums, and because I was unsure that there could be a complete theoretical solution, I decided to embark on a series of experiments. Having made some tentative enquiries about research grants amongst the horological institutions (this was quite a few years ago) I finally decided to carry out the experiments myself.

Wood for the bobs offered the best compromise in cost and ease of manufacture. As to the wood-turning I was very fortunate in knowing of the craft workshops in the nearby Broadmoor Hospital, and I am indebted to their staff and the craftsman who diligently made the bobs in batches over an 18 month period. The standard of turning is excellent and conforms to the specified dimensions within a millimetre in most cases. The polishing was done by myself.

A source of encouragement came from the late Kenneth James, an able engineer and mathematician who did some of the necessary calculations.

One of the starting points is that all the bobs should be of the same mass and volume as a 10cm diameter sphere (the reference shape). The 10cm dimension was chosen as being representative of 'typical' pendulum dimensions, and if made of lead would give a mass of nearly 6kg. The other starting point is that the bobs were all hung as 1m pendulums beating seconds within experimental error.

Thirdly, the bob shapes were made in definite families that differ from one another in simple ratios of length (or thickness) to diameter. For example the lenticular bobs varied from the fat 1:2, to the fairly conventional 1:4. The resultant shapes are shown in the colour illustrations: all have the same mass and volume to within a few per cent.

In order to measure the losses, the bobs had to be swung with a bifilar suspension. Two long fine wires were needed to keep the bob in the plane of the swing and the suspension frame was cross shaped, rather like the frame for manipulating a marionette. The suspension wires, only 0.006in (0.15mm) diameter piano wire, were the finest that would reliably support the bobs and allow some adjustment for length and balancing. Small taper pins held the wires in place in the bobs. A subsidiary series of tests showed that the drag of the pendulum rods would be quite significant, and even the drag of two fine wires is quite detectable.

The amplitude of swing was determined with an optical lever system. A lens and mirror gave an autocollimation of a lamp filament on to an adjacent screen that could be marked and so give a permanent record - see the drawing on page 36. The suspended bobs were hung well away from a wall to avoid any air flow interaction with the wall.

The loss measurements were made by setting the bob swinging to about 2½ degrees amplitude, waiting for any wobbles to die away, and then measuring the amplitude decay for 16 data points at one minute invervals, covering the range 2 degrees to about 1 degree. The decay measurements were all 'linear' (ie true damped harmonic motion within experimental error). The mathematical best fit to give the damping coefficient all had regression coefficients better than 0.999. The repeatability was good - over eight measurements on the sphere the range was 3 per cent; the standard deviation 2 per cent.

The results are summarised in the table on page 37 using a point system to show relative merit with the sphere assigned the value 100. These results include corrections for small differences in mass and volume. Note that the apparent fine detail is not entirely justified in terms of experimental error, but is given where there is a trend within a family of shapes.

All of the above results are for the bobs hung on two fine wires. For the special case of the sphere a single wire will suffice and the air resistance reduces by a considerable 11 per cent. For those readers interested in the energy losses, the essential facts are that the sphere on two wires had a Q of 2950 and a mass of 380g. (Considering that the bob is made of wood, the Q is quite high. If made of lead it would have the exceptionally good value of 46,000; tungsten alloy would be better still at 73,000.)

Conclusions

A sphere is a very good shape. Only a handful of shapes are better, but the sphere regains nearly all the advantage if hung from a single wire. On the other hand, research into a more aerodynamic rod shape could help to retain the benefits of the parabolic spindles. A thin flat rod of elliptical cross section may be the best.

Quite a practical shape is the cylinder with hemispherical ends with its long axis in the direction of swing. By analogy with the parabolic spindles, a 2:1 length to diameter is probably a reasonable compromise.

The lenticular shapes have rapidly increasing surface areas (the 4:1 shape with an area of 500 cm² has the largest area of all the shapes measured) and are not the best shapes. They are, however, aesthetically pleasing and probably the best compromise in a narrow clock case or if they have to swing close to the back board.

One result worth mentioning is the surprisingly good figure of merit for the double truncated cone when compared with its near neighbour, the 1:3 lenticular shape. Another significant practical result ▷

was obtained with an identical double truncated cone, but with a rounded edge. When the bob was being made a knot appeared in the surface of the turned wood. The craftsman set this bob to one side and made another! I took the "reject" shape (not shown in the photographs) and rounded the complete rim so that it was shaped like a discus. With the rounded edge the figure of merit was identical, within experimental error, to the sharp-edged version. We may therefore conclude that a sharp edge is not strictly necessary on a lenticular bob.

The best shapes are the unusual parabolic spindles. The outline is obtained by the revolution of part of a parabola, $y = x^2$, about a line parallel to the x axis. Kenneth James and I chose this form as a compromise between a known ideal in viscous flow and to approach the early 'pickaxe' design by Huygens. In fact the 2:1 shape may well be the best shape possible as a theoretical analysis of a shape to give minimum drag in purely viscous flow (sometimes known as Stokes Flow, after Sir George Stokes who investigated the internal friction of air in the 1850s). The optimum shape is very similar to the short parabolic spindle, with a length to diameter 1.95:1. The only other difference is that the optimum shape has a tip with an included angle of 120 degrees, whereas the experimental bob has an angle of about 105 degrees. The theoretical analysis is by O Pironeau, 'On Optimum Profiles in Stokes Flow', *Journal of Fluid Mechanics*, 1973, Vol 59, pp 117-128.

The Aked errors

Referring to Charles Aked's article in the May issue of *Clocks*, let us examine some of his statements and false conclusions.

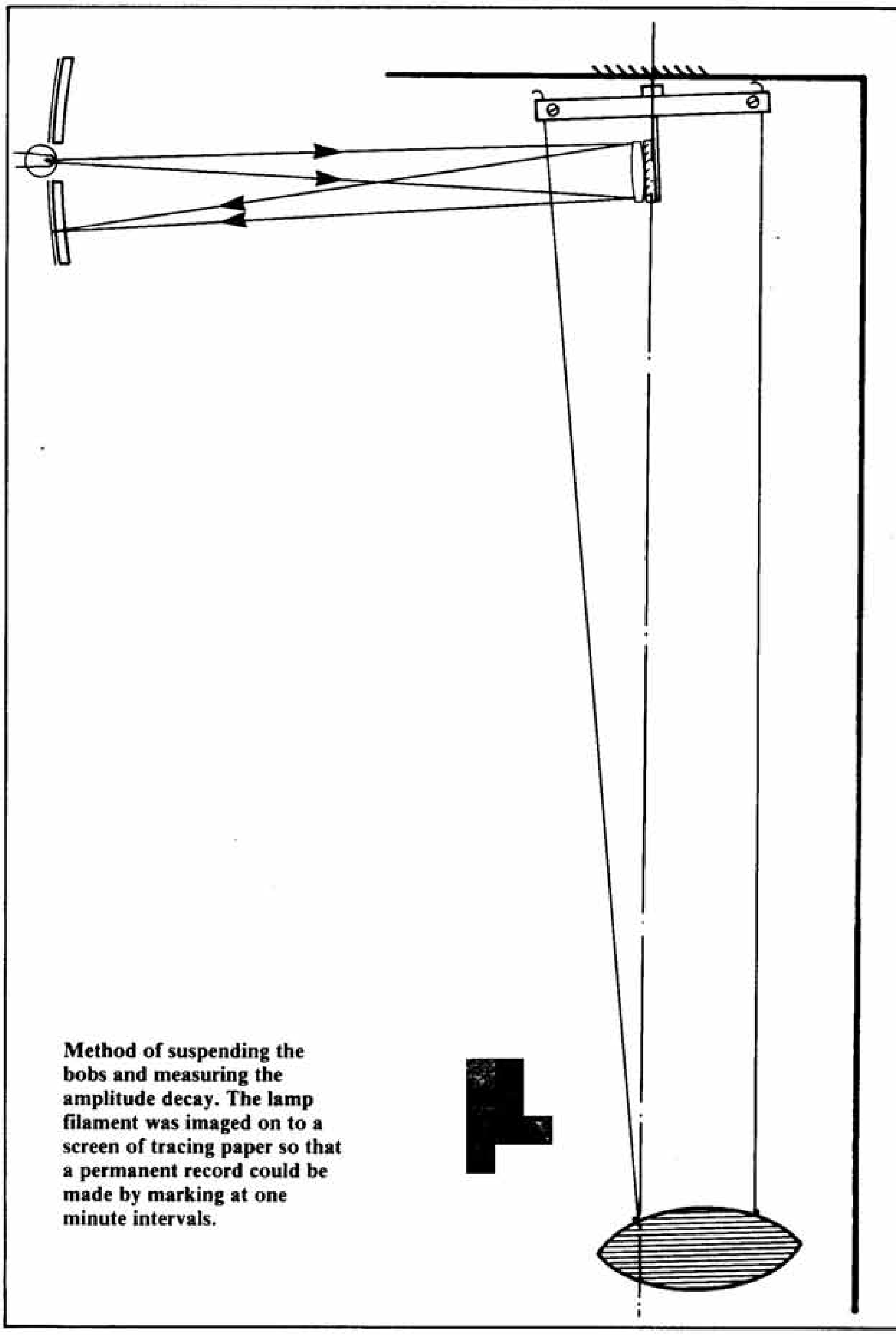
Choice of materials: Mr Aked has jumped from mercury to gold and platinum, and not surprisingly, he ruled out precious metals due to cost. He has overlooked tungsten and commercial uranium. Tungsten is widely used in lamp filaments, munitions, machine tool boring bars which are 'chatter-free' and in the bodies of darts made for the game of darts. Unmachined bar costs about £25 per kg – not excessive when compared to the man-hours that an enthusiast will put into the making of a clock. The relative densities of tungsten alloys range from 16 to 18. Commercial uranium (sometimes called depleted uranium) has a relative density of 18.8. It is available from British Nuclear Fuels in unmachined billets for about £9 per kg. Unfortunately, the slight radioactivity and certain machining problems make it more applicable to a laboratory than a domestic clock.

Friction of air: Mr Aked has neglected to take this into account. From my reading of books on aerodynamics and fluid dynamics, the speed of a pendulum bob is such that the viscous drag and form drag are comparable.

Volume of air moved each swing: Any shape will carry some air with it and is known as the virtual mass or added mass phenomena. A rule of thumb in fluid dynamics is that a circular cylinder will carry about the same volume in air as the cylinder, the sphere about half, and the parabolic spindle very much less than half. NB: This added mass is the main cause of the increase in barometric error in pendulum clocks when compared with the theoretical value due to the flotation of the bob alone.

Projected area: This is too simplistic – the detailed shape of the bob is much more important. If we look at the results for the circular cylinder, then we see that both the 3:1 and 2:1 cylinders are indeed better with the long axis in the direction of swing than in the usual way, but for the 1:1 shape the situation is reversed. The dominant effect is almost certainly due to the sharp edges, because if these are turned into curves we get more dramatic improvements. Again the benefits of curved surfaces are seen with the cylinder domed at one end and at both ends. Note also that the narrow 3:1 spindle is worse than its fatter relative.

The projected area argument can be taken to the limit by making the bob into a hoop which is heavier in one region. It will 'pump' no air at all, and has no face to present 'into the wind'. Unfortunately the surface area will be considerable. If we take, purely as an example, Mr Aked's volume of iron as an 80cm long slug in a 3cm tube 2m in diameter, so that it would



Method of suspending the bobs and measuring the amplitude decay. The lamp filament was imaged on to a screen of tracing paper so that a permanent record could be made by marking at one minute intervals.

oscillate like a weighted bicycle wheel, the resulting torus will have an area of about 6000 sq cm.

Surface Finish: Having consulted some professional fluid dynamicists I have been advised that at the typical maximum velocities of a pendulum bob, the boundary layer will be about 3mm deep. The boundary layer is defined as the region of velocity gradient between air on the surface of the bob moving at bob velocity and the distance to almost still air. Surface finish is not, therefore, expected to be critical. The situation with clock pendulums is *not* to be compared with the surface of a golf ball where the dimples have a profound effect on reducing the drag when it reaches speeds of between 125-200 miles per hour!

Reduction of air pressure: The viscosity of air is independent of pressure (see *Shape and Flow, the Fluid Dynamics of Drag*, by A H Shapiro, Heinemann) so attention to shape will still be beneficial even at quite low pressures.

Rank order of shapes: Despite referring to my experiments (I have lectured on the results several times) Mr Aked has drawn conclusions which are almost the exact opposite of the experimental findings. The only exception is the tall circular cylinder.

Torsional pendulum: I reported on the losses of the Atmos torsional pendulum in my series on 'Vibration Theory and Clocks' in *Horological Journal*, September 1977. The power required to maintain the pendulum at its working amplitude is only 0.06 microwatts yet the quality factor is a poor 330. In other words, the ratio of stored energy to energy losses is low, lower even than a restless 'hi-beat' watch balance. The Q and power requirements of the Atmos ring could, in my opinion, be improved considerably by reducing the surface area and using high density metals.

Overall conclusions

The experiments with a variety of bob shapes were conducted very carefully, using realistic workings amplitudes, a massive support, and very thin suspension springs. I believe the orders of merit to be correct and to form a sound basis for comment on pendulum design.

The first conclusion is that certain shapes, such as the sphere, rugby ball shape and similar shapes offer significant reductions in air resistance. In the interests of precision timekeeping the benefits can be shown as:

a. A direct saving of an expensive metal
b. An increase in Q
c. A reduction in power requirements from the train.

However, the differences between the best and the worst (excluding ornamental designs) is not large. Not large enough, that is, to be obvious in comparative time-keeping nor, unfortunately, in the different power requirements. It must be said therefore, that the horological evolutionary forces are not strong in respect of bob shapes. Nevertheless, some clocks have been made with 'good' bobs. In H Alan Lloyd's book *The Collector's Dictionary of Clocks*, Country Life Books, 1964, there is a photograph on page 158 showing 'an Austrian skeleton clock, c1870 (Mr W Pinder)' with a bob similar to the 3:1 parabolic spindle. Outlines on the hands echo the shape of the bob. (It will be interesting to know the current location of this clock.)

Eli Terry experimented with various clock designs, one of which incorporated a beautifully polished lead-filled brass-cased 'football' bob of about 3:1 length to diameter – see 'An Eli Terry Regulator' by Elmer C Korten, NAWCC *Bulletin* 436-453, Number 159, August 1972. I am indebted to Pierre Boucheron for this reference.

A Thomas Cole clock was for sale recently with a spherical bob – see *Clocks*, page 29, Volume 10, February 1988.

A spherical pendulum is featured in the magnificent table regulator by Sinclair Harding and Co. When the clock was announced in *Clocks*, April 1986, I contacted their office to enquire about their statement about featuring 'a high-Q pendulum'. I was lucky to speak to Mike Harding and he told me that he had followed many of my articles, including a meeting report on one of my lectures on the bob shapes. It is extremely gratifying that the research has influenced the designers to lead to such an elegant pendulum. Based on the experiments I estimate that the pendulum will have a Q in excess of 20,000 – twice as good as many conventional regulators.

A pendulum with the prolate spheroid shaped bob has been made by a transatlantic correspondent, Pierre Boucheron. Boucheron has provided invaluable data for research on pendulum precision time-keeping by reactivating the Shortt clock Number 41 at the United States Naval Observatory, Washington, and conducting his own experiments into knife-edge suspensions. Noting that an Indian brass vase (an 'Inja' vase) contained the essential outline for an ellipsoidal bob (prolate spheroid) he filled the vase with 22lbs of lead and obtained a Q of 26,000, more than double that of many regulators.

Clocks contributor, Laurie Penman, made a clock for the Exeter Museum in which he turned the circular cylinder on its side. This gives a rather hammer-head style pendulum, and from my measurements, will achieve a 30 per cent increase in Q and a 30 per cent reduction in power requirements for the same amplitude.

My advice to designers is clear and very simple – design the bob to suit your own practical and aesthetic requirements, aiming to use high density materials, the minimum surface area, and avoiding sharp corners.

In conclusion, there is no doubt that the sphere is a good shape, only exceeded by a rugby ball and similar unusual shapes. There is one shape that is likely to be the very best and that is the 2:1 parabolic spindle. ☐

Figures of merit for different bob shapes

Shape	**Ratio** (length:diameter)	**Merit**	**Surface area** (cm²)
Parabolic spindle	2:1 H	115	334
Parabolic spindle	3:1 H	114	365
Parabolic spindle	4:1 H	106	395
Prolate spheroid	2:1 H	114	338
Sphere	—	100	314
Oblate spheroid	1:2 D	98	344
Double truncated cone (cf Riefler)	1:3 D	100	416
Lenticular	1:2 D	97	358
Lenticular	1:3 D	85	429
Lenticular	1:4 D	72	502
Circular cylinder	3:1 V	64	403
Circular cylinder	3:1 H	83	403
Circular cylinder	2:1 V	68	378
Circular cylinder	2:1 H	78	378
Circular cylinder	1:1 V	75	360
Circular cylinder	1:1 H	62	360
Circular cylinder	1:2 D	79	381
Circular cylinder	1:3 D	77	416
Circular cylinder	1:4 D	70	453
Circular cylinder with single hemispherical end (cf Big Ben)	2:1 V	73	360
Circular cylinder with two hemispherical ends	2:1 V	79	341
Circular cylinder with two hemispherical ends	2:1 H	108	341

Orientation
H long axis in direction of swing
V long axis vertical
D diameter in vertical plane and direction of swing